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Visualization of Natural Convection in a Vertical Annular Cylinder with a Partially Heat Source and Viscous Dissipation

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Abstracts

In this work, we visualize the effect of viscous dissipation parameter(ε) on the heat transfer by supplying the heat at three different positions to the vertical annular cylinder embedded with porous medium. Finite element method has been used to solve the governing equations. Influence of Aspect Ratio(A_r), Radius Ratio(R_r) on Nusselt number $(\overline{Nu})$ is presented. The effect of viscous dissipation parameter(ε), for different values of Rayleigh number is discussed. The fluid flow and heat transfer is presented in terms of streamlines and isotherms.

Keywords: Natural Convection, Porous medium, viscous dissipation parameter(ε), Aspect Ratio(A_r), Radius Ratio(R_r) and Rayleigh number(Ra)

Introduction

The viscous dissipation effect, which is a local production of thermal energy through the mechanism of viscous stresses, is a ubiquitous phenomenon and it is encountered in both the viscous flow of clear fluids and the fluid flow within porous media. When compared with other thermal influences on fluid motion (i.e., by means of buoyancy forces induced by heated or cooled walls and by localized heat sources or sinks) the effect of the heat released by viscous dissipation covers a wide range of magnitudes from being negligible to being significant.

Gebhart [1] discussed this range at length and stated that “a significant viscous dissipation may occur in natural convection in various devices which are subject to large decelerations or which operate at high rotational speeds. In addition, important viscous dissipation effects may also be present in stronger gravitational fields and in processes wherein the scale of the process is very large, e.g., on Larger planets, in Large masses of gas in space, and in Geological processes in fluids internal to various bodies.” In contrast to such situations, many free convective processes are not sufficiently vigorous to result in a significant quantitative effect, although viscous dissipation sometimes serves to alter the qualitative nature of the flow.

Although viscous dissipation is generally regarded as a weak effect, a property it shares with relativistic and quantum mechanical effects in everyday life, it too has played a seminal role in history of physics. It was precisely this “weak” physical effect that allowed James Prescott Joule in 1843 to determine the mechanical equivalent of heat using his celebrated paddle-wheel experiments, and thereby to set in place one of the most important milestones towards the formulation of the first principle of thermodynamics.

There is an increasing interest in the study of natural convection in fluid saturated porous media as proved by the explosive growth in the literature on the subject and also an increasing interest in the consideration of the viscous dissipation effects on the flow and temperature fields as well as on the heat transfer performance of the involved devices. From an order of magnitude analysis it can be concluded that the viscous dissipation can be neglected in many situations of practical interest both for domains filled with a clear fluid or for domains filled with fluid-saturated porous media. This is, however, a subject that attracts many researchers and in particular special attention is being devoted to the natural convection in enclosures filled with a fluid-saturated porous medium including the viscous dissipation effects.

Going on to the literature, one can find many recent works concerning the natural convection in fluid-saturated porous media including viscous dissipation effects. Examples of work considering the Darcy Law to describe the fluid flow are these of Nakayama and Pop [2] Magyari and Keller [3], Rees et al. [4], Saied and Pop [5] and Rees [6]. In the work of Al-Hadhrami et al. [7] it is considered the Brinkman extension of the Darcy Law and a quadratic drag term on the momentum equation is considered in the works of Murthy and Singh [8], Murthy [9], Tashtoush [10] and Magyari et al. [11].

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The book by Nield and Bejan [12] gives a very good description about the relevance of the subject of heat transfer in porous media and about the models used to take into account the different effects on the natural convection in fluid-saturated porous media.

Nield [13] gives an explanation why the quadratic drag term on the momentum equation (which does not contain the viscosity in an explicit way) must be taken into account as a dissipation term. A study of the entropy generation associated with the natural convection heat transfer problem in a inclined square enclosure filled with a fluid-saturated porous medium was conducted by Baytas [14]. In this work, the viscous dissipation term is not taken into account in the energy conservation equation but it is taken into account in the entropy generation equation. Israel et al [15] have studied the influence of viscous dissipation and radiation on unsteady free convection flow past an infinite vertical plate in a porous medium with time-dependent suction. They discussed the effect of material properties on the temperature and velocity profile. They discussed the effect of material properties on the temperature and velocity profile. They conclude that the increased viscous dissipation leads to increased temperature profile, increase in the magnetic field decreases the temperature profile on cooling and increased cooling of the plate and viscous dissipation results in increased velocity profile. Saied [16] also studied the effect of viscous dissipation on free convection in a porous cavity and presented results in terms of local and average Nusselt number at vertical hot and cold walls of the cavity. Abo-Eldahab and Aziz [17], analyzed the effects of viscous and Joules heating on MHD free convection flow past a semi-infinite vertical flat plate in the presence of the combined Hall and ion-slip currents for the case of power-law variation of the wall temperature. Viscous and Joule heating effect on forced convection flow of ionized gases adjacent to isothermal porous surfaces is analyzed numerically by Duwairi[18]. He found that heat transfer rate is decreased due to viscous dissipation effect in both the cases of suction or injection in the fluid.

The present study focuses on the effect of viscous dissipation on the heat transfer behavior in a saturated porous medium embedded in a vertical annular cylinder by supplying the heat at three different locations of the vertical annular cylinder. Finite Element Method has been used to convert the partial differential equations into a matrix form of equations, which can be solved iteratively with the help of a computer code. The Galerkin Finite Element Method of three nodded triangular elements is used to divide the physical domain into smaller segments, which is a pre-requisite for finite element method. Influence of Aspect Ratio (A_r), Radius Ratio (R_r), Viscous dissipation parameter (ε) and Rayleigh number on the average Nusselt number ($\overline{Nu}$) is presented. The fluid flow and heat transfer is presented in terms of streamlines and isotherms.

Nomenclature :

A_r : Aspect ratio
C_p : Specific heat
D_p : Particle diameter
g : Gravitational acceleration
H_t : Height of the vertical annular cylinder
K : Permeability of porous media
L : Length
P : Pressure
$\overline{Nu}$: Average Nusselt number
q_t : Total heat flux
r,z : Cylindrical co-ordinates
$\bar{r}, \bar{z}$: Non-dimensional co-ordinates
r_i, r_o : Inner and outer radius
Ra: Rayleigh number
R_r : Radius ratio
T : Temperature
$\bar{T}$: Non-dimensional Temperature
u : Velocity in r direction
w : Velocity in z direction
x,y : Cartesian co-ordinates
$\bar{x}, \bar{y}$: Non-dimensional co-ordinates

Greek Symbols:

α : Thermal diffusity
β_T : Coefficient of thermal expansion
ε : Viscous dissipation parameter
ΔT : Temperature difference
σ : Stephan Boltzmann constant
ρ : Density
ν : Coefficient of kinematic viscosity
μ : Coefficient of dynamic viscosity
φ : Porosity
ψ : Stream function
$\bar{\psi}$: Non-dimensional Stream function

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Subscripts:
w : Wall

$_\infty$: Conditions at infinity

Mathematical analysis

A vertical annular cylinder of inner radius r_i and outer radius r_o as depicted by schematic diagram as shown in figure (A) is considered to investigate the heat transfer behavior. The co-ordinate system is chosen such that the r-axis points towards the width and z-axis towards the height of the cylinder respectively. Because of the annular nature, two important parameters emerge which are aspect ratio, A_r and radius ratio R_r of the annulus. They are defined as $A_r = \frac{H_t}{r_o - r_i}$, $R_r = \frac{r_o - r_i}{r_i}$, where H_t is the height of the annular cylinder. The inner surface of the cylinder is maintained at isothermal temperature T_h and outer surface is at ambient temperature T_∞. It may be noted that, due to axisymmetry only half of the annulus is sufficient for analysis purpose, since other half is mirror image of the first half. The top and bottom horizontal surfaces of the vertical annular cylinder are adiabatic.

The flow inside the porous medium is assumed to obey Darcy law and there is no phase change of fluid. The properties of the fluid and porous medium are homogeneous, isotropic and constant except variation of fluid density with temperature. The fluid and porous medium are in thermal equilibrium. With these assumptions, the governing equations are given by

Continuity Equation: $\frac{\partial (ru)}{\partial r} + \frac{\partial (rw)}{\partial z} = 0$ ……….(1)

The velocity in r and z directions can be described by Darcy law as:

velocity in horizontal direction: $u = \frac{-K}{\mu}\frac{\partial p}{\partial r}$

velocity in vertical direction: $v = \frac{-K}{\mu}\left(\frac{\partial p}{\partial z} + \rho g\right)$

The permeability K of porous medium can be expressed as Bejan [19]

$$K = \frac{D_p^2 \phi^3}{180(1-\phi)^2}$$

Momentum Equation: $\frac{\partial w}{\partial r} - \frac{\partial u}{\partial z} = \frac{gK\beta}{\nu}\frac{\partial T}{\partial r}$ ……. (2)

Energy Equation: $u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z} = \alpha\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right) + \frac{\mu}{K(\rho C_p)_f}(u^2 + w^2)$ ….(3)

The continuity equation (1) can be satisfied by introducing the stream function ψ as

$$u = -\frac{1}{r}\frac{\partial \psi}{\partial z}, \quad \text{…….. (4)}$$

$$w = \frac{1}{r}\frac{\partial \psi}{\partial r} \quad \text{……… (5)}$$

The variation of density with respect to temperature can be described by

Boussinesq approximation as $\rho = \rho_\infty[1 - \beta_T (T - T_\infty)]$ ……… (6)

The corresponding boundary condition, when heat is supplied at three different locations at the inner wall of the vertical annular cylinder:

At $r = r_i$ and $0 \le z \le \frac{H}{6}, \frac{5H}{12} \le z \le \frac{7H}{12}, \frac{5H}{6} \le z \le H, T_w = T_\infty + B(z)^\lambda, \psi = 0$

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At $r = r_0$ $\quad T = T_\infty, \psi = 0$

The new parameters arising due to cylindrical co-ordinates system are

Non-dimensional Radius $\bar{r} = \frac{r}{L}$ ……. (7)

Non-dimensional Height $\bar{z} = \frac{z}{L}$ ……. (8)

Non-dimensional Stream function $\bar{\psi} = \frac{\psi}{\alpha L}$ ……. (9)

Non-dimensional Temperature $\bar{T} = \frac{(T - T_\infty)}{(T_W - T_\infty)}$ ……. (10)

Rayleigh Number $Ra = \frac{g\beta_T \Delta TKL}{\nu\alpha}$ ……. (11)

Viscous dissipation parameter $\varepsilon = \frac{\alpha\mu}{\Delta TK\rho C_p}$ ……. (12)

The non-dimensional equations for the heat transfer in vertical cylinder are

Momentum equation: $\frac{\partial^2\bar{\psi}}{\partial\bar{z}^2} + \bar{r}\left(\frac{1}{\bar{r}}\frac{\partial\bar{\psi}}{\partial\bar{r}}\right) = \bar{r}Ra\frac{\partial\bar{T}}{\partial\bar{r}}$ …….. (13)

Energy Equation:

$$\frac{1}{\bar{r}}\left[\frac{\partial\bar{\psi}}{\partial\bar{r}}\frac{\partial\bar{T}}{\partial\bar{z}} - \frac{\partial\bar{\psi}}{\partial\bar{z}}\frac{\partial\bar{T}}{\partial\bar{r}}\right] = \left(\frac{1}{\bar{r}}\frac{\partial}{\partial\bar{r}}\left(\bar{r}\frac{\partial\bar{T}}{\partial\bar{r}}\right) + \frac{\partial^2\bar{T}}{\partial\bar{z}^2}\right) + \varepsilon\left[\left(\frac{1}{\bar{r}}\frac{\partial\bar{\psi}}{\partial\bar{r}}\right)^2 + \left(\frac{1}{\bar{r}}\frac{\partial\bar{\psi}}{\partial\bar{z}}\right)^2\right] \quad \text{……. (14)}$$

The corresponding non-dimensional boundary conditions, when heat is supplied at three different locations at the inner wall of the vertical annular cylinder

At $r = r_i$ and $0 \le z \le \frac{H}{6}, \frac{5H}{12} \le z \le \frac{7H}{12}, \frac{5H}{6} \le z \le H, \bar{T} = T_\infty + B(z)^\lambda, \bar{\psi} = 0$

At $r = r_0$ $\quad \bar{T} = 0,\ \bar{\psi} = 0$

Method of solution

Equations (13-14) are coupled partial differential equations to be solved in order to predict the heat transfer behavior. These equations are solved by using finite element method. Galerkin approach is used to convert the partial differential equations into a matrix form of equations. A simple 3 noded triangular element is considered. The polynomial function for "T" can be expressed as

$$T = \alpha_1 + \alpha_2 r + \alpha_3 z \quad \text{……… (15)}$$

The Variable T has the value T_i,T_j and T_k at the nodal position i, j and k of the element. The r and z co-ordinates at these points are r_i, r_j, r_k and z_i, z_j, z_k respectively.

since $T = N_i T_i + N_j T_j + N_k T_k$ ………. (16)

where N_i, N_j & N_k are shape functions given by

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$$N_m = \frac{a_m + b_m r + c_m z}{2A}, \qquad m=1,2,3 \qquad \text{………. (17)}$$

and a_m, b_m, c_m are matrix coefficients

Integrating equations (13-14) using Galerkin method the Momentum equation leads to

$$\frac{2\pi\bar{R}}{4A}\left[\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix} + \begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2 \end{bmatrix}\begin{Bmatrix}\bar{\psi}_1\\ \bar{\psi}_2 \\ \bar{\psi}_3\end{Bmatrix}\right]$$

$$= -\frac{2\pi\bar{R}^2 Ra}{6}\begin{Bmatrix} b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \\ b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \\ b_1\bar{T}_1 + b_2\bar{T}_2 + b_3\bar{T}_3 \end{Bmatrix} \qquad \text{………. (18)}$$

The Stiffness matrix of Energy equation is given by

$$\left[\frac{2\pi}{12A}\begin{Bmatrix} c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \\ c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \\ c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3 \end{Bmatrix}\begin{bmatrix} b_1 & b_2 & b_3 \end{bmatrix} - \frac{2\pi}{12A}\begin{Bmatrix} b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \\ b_1\bar{\psi} + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \\ b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3 \end{Bmatrix}\begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix}\right]\begin{Bmatrix} T_1 \\ T_2 \\ T_3 \end{Bmatrix}$$

$$+\frac{2\pi\bar{R}}{4A}\left\{\begin{bmatrix} b_1^2 & b_1b_2 & b_1b_3 \\ b_1b_2 & b_2^2 & b_2b_3 \\ b_1b_3 & b_2b_3 & b_3^2 \end{bmatrix}\begin{Bmatrix}\bar{T}_1\\ \bar{T}_2 \\ \bar{T}_3\end{Bmatrix} + \begin{bmatrix} c_1^2 & c_1c_2 & c_1c_3 \\ c_1c_2 & c_2^2 & c_2c_3 \\ c_1c_3 & c_2c_3 & c_3^2 \end{bmatrix}\begin{Bmatrix}\bar{T}_1\\ \bar{T}_2 \\ \bar{T}_3\end{Bmatrix}\right\}$$

$$+\frac{2\pi A\varepsilon}{12\bar{r}}\left[\begin{bmatrix}1\\1\\1\end{bmatrix}\left[b_1\bar{\psi}_1 + b_2\bar{\psi}_2 + b_3\bar{\psi}_3\right]^2 + \begin{bmatrix}1\\1\\1\end{bmatrix}\left[c_1\bar{\psi}_1 + c_2\bar{\psi}_2 + c_3\bar{\psi}_3\right]^2\right] = 0 \qquad \text{………. (19)}$$

1. ***Results and Discussion:***

Results are obtained in terms of Nusselt number at hot wall for various parameters such as viscous dissipation parameter "ε", Aspect Ratio(A_r), Radius ratio(R_r) and Rayleigh number(Ra), when heat is supplied at three different locations of the hot wall of the vertical annular cylinder.

The average Nusselt number is given by $\overline{Nu} = -\frac{1}{L}\int_0^L \left(\frac{\partial\bar{T}}{\partial\bar{r}}\right)_{\bar{r}=r_i,r_0}$ ….. (20)

Where L is the length of the heated wall of the vertical annular cylinder. $i.e., L = L_1 + L_2 + L_3$

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a)

b)

c)

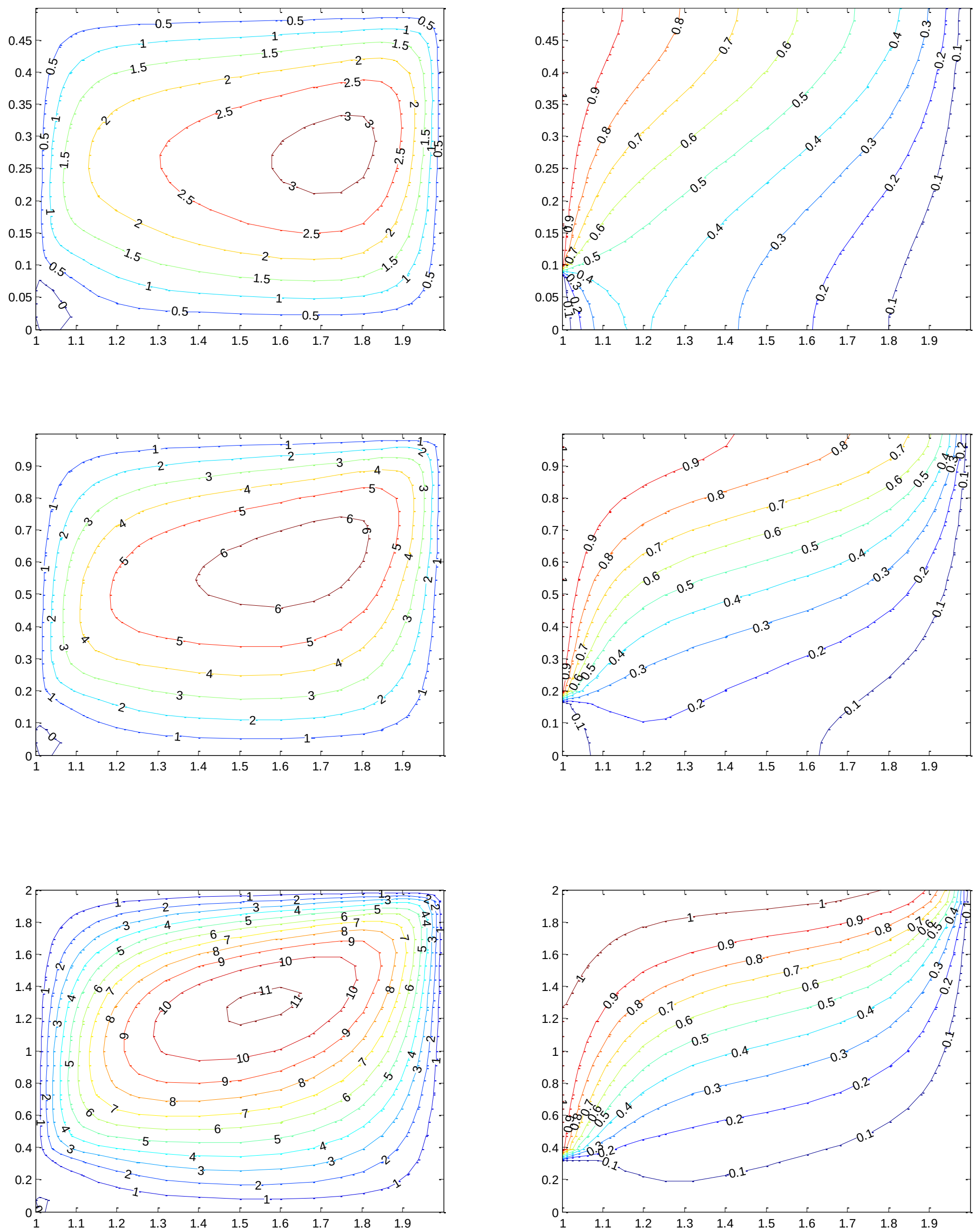

Fig:1: Streamlines(left) and Isotherms(Right) for ε=0.01, R_r =1, Ra=100 A_r =0.5 b) A_r =1 c) A_r =2

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a)

b)

c)

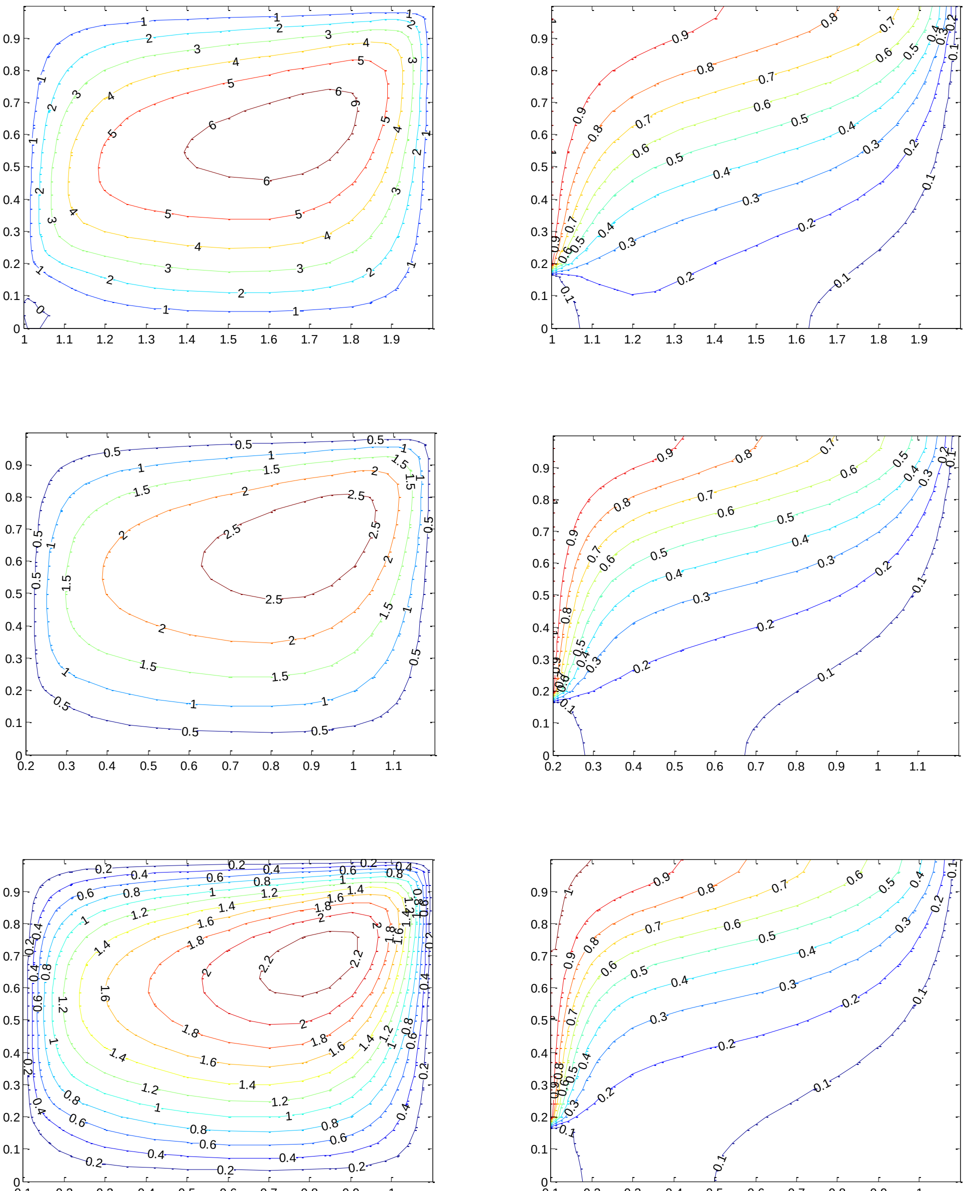

Fig:2: Streamlines(left) and Isotherms(Right) for ε=0.01, A_r =1, Ra=100 a) R_r =1 b) R_r =5 c) R_r =10

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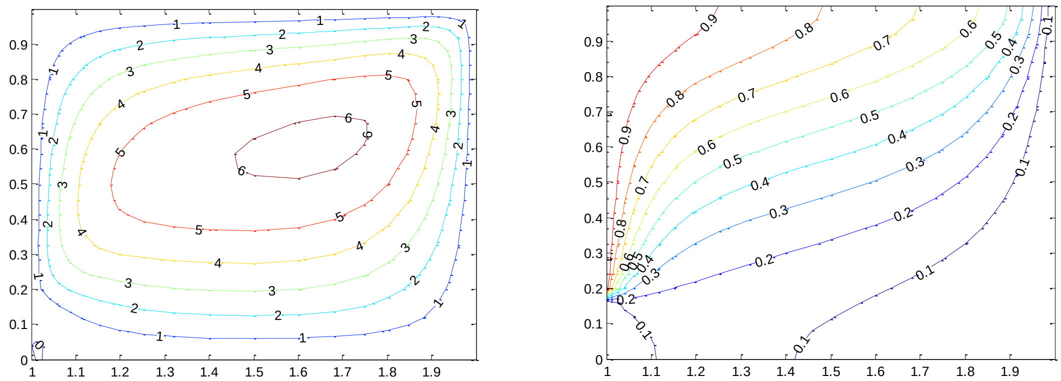
a)



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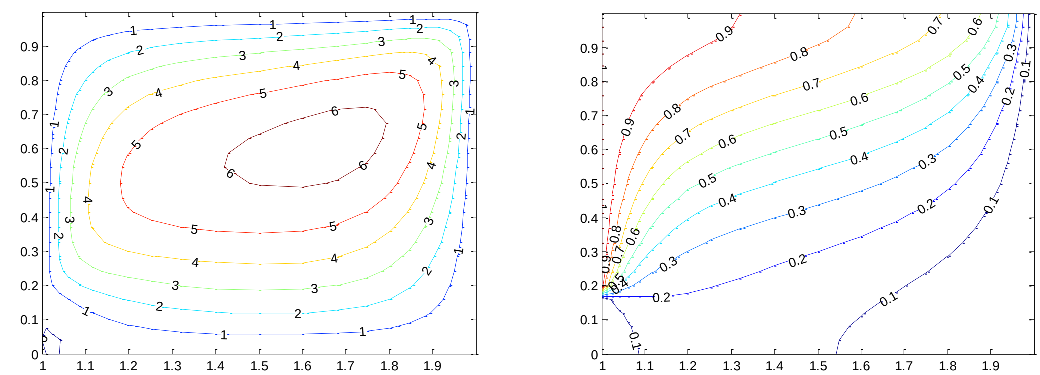

c)

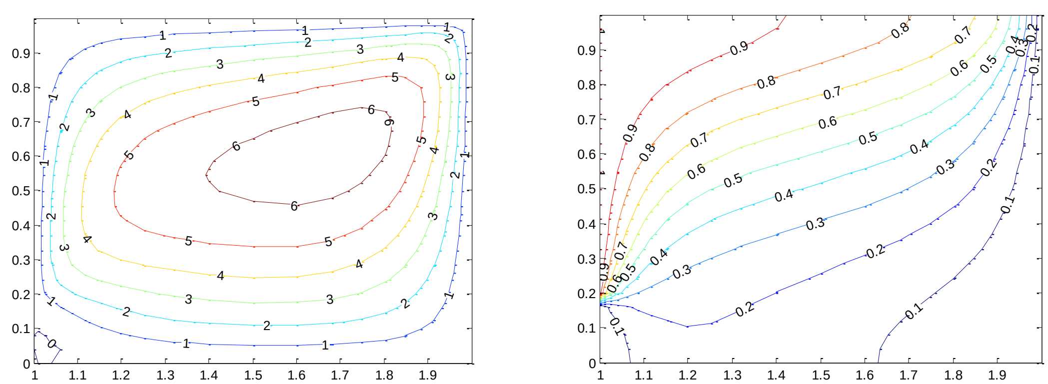

Fig:3: Streamlines(left) and Isotherms(Right) for A_r =1, R_r =1, Ra=100 a)ε=0 b) ε=0.005 c) ε=0.01

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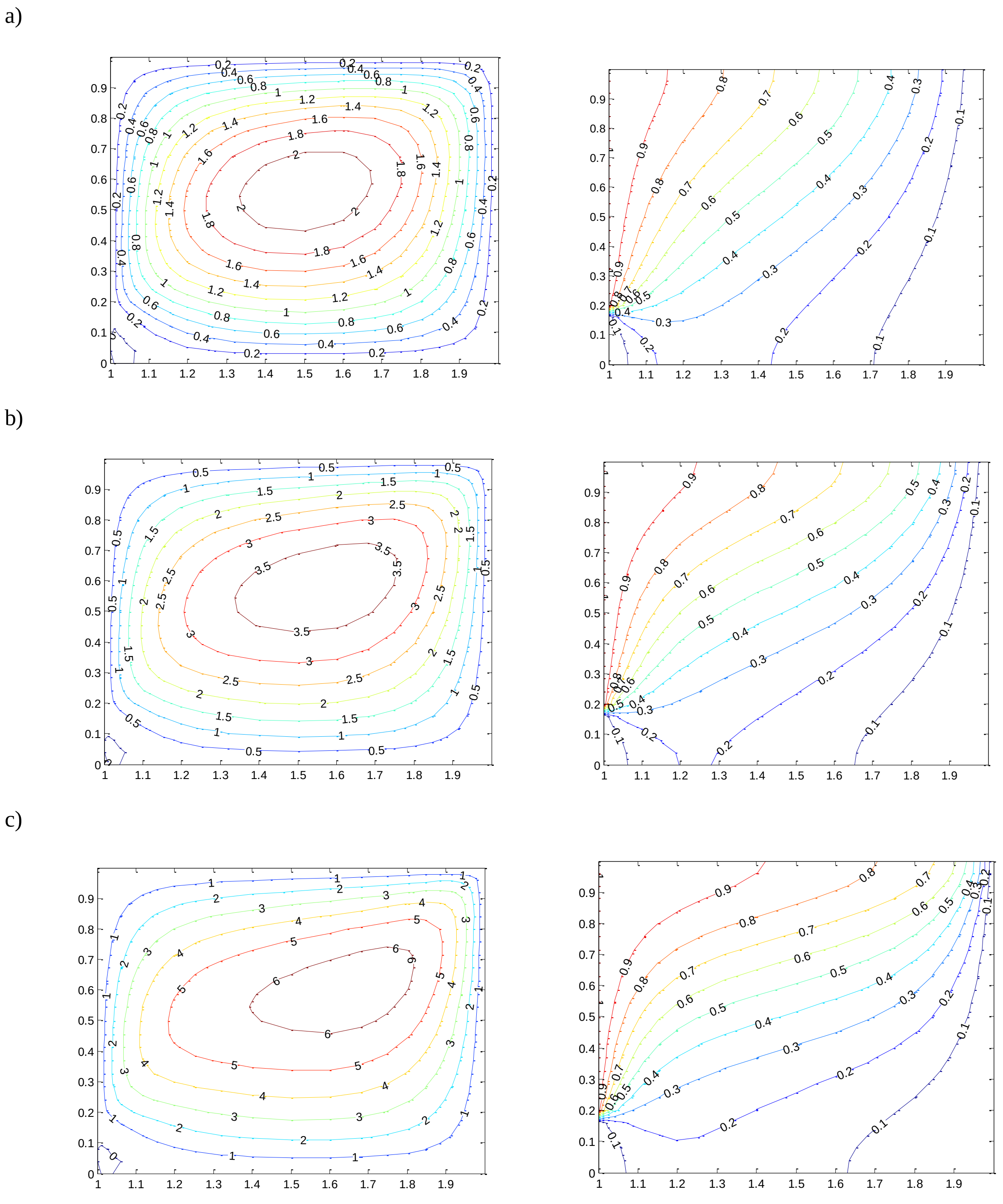

Fig:4: Streamlines (left) and Isotherms(Right) for A_r =1, R_r =1, ε=0.01a) Ra=25 b) Ra =50 c) Ra=100

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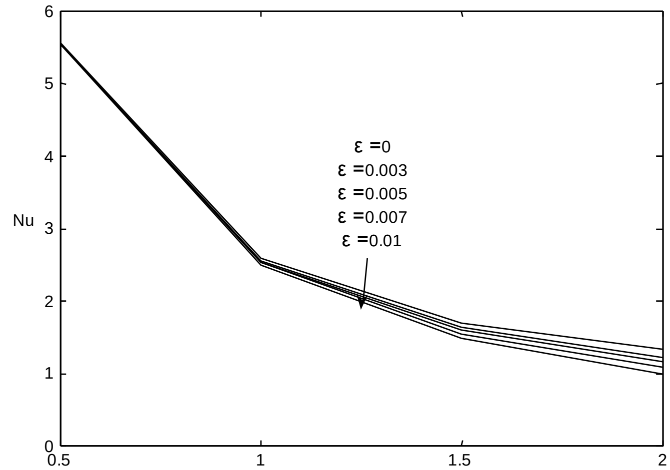
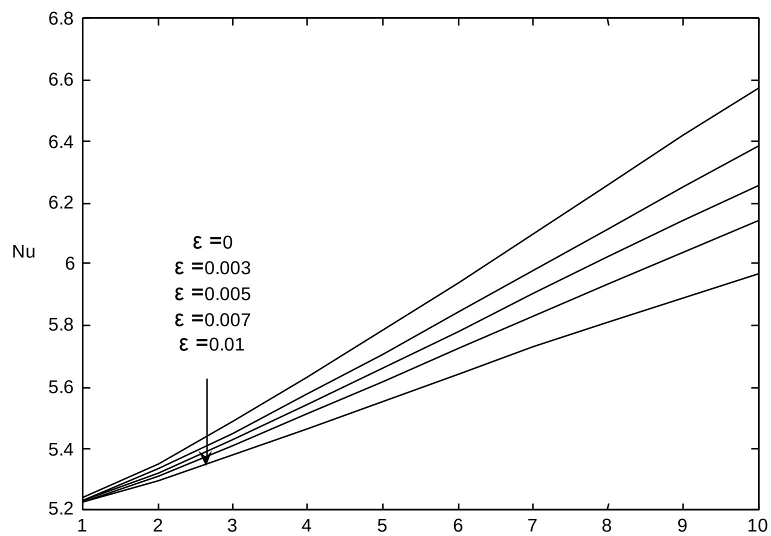


Aspect Ratio
Fig:5: Nu Variations with A_r at hot surface for different values of ε at R_r=1, Ra=50



Radius Ratio
Fig:6: Nu Variations with R_r at hot surface for different values of ε at A_r =0.5, Ra=100

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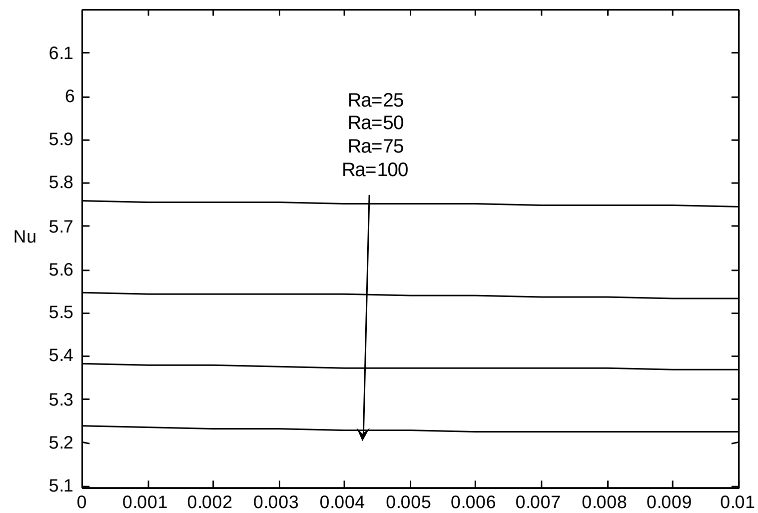

Viscous Dissipation Parameter (ε)
Fig:7: Nu Variations with λ at hot surface for different values of Ra at A_r***=0.5,*** R_r***=1***

Fig:1 shows the streamlines and isothermal lines distribution inside the porous medium with respect to various values of Aspect Ratio(A_r). The streamlines and isothermal lines shift from the lower portion of the hot wall to the upper portion of the cold wall as Aspect Ratio(A_r) increases. So, the circulation of the fluid covers the whole domain at lower values Aspect Ratio(A_r) as compared to the higher values of Aspect Ratio(A_r). It is clearly observed that high convection heat transfer takes place at upper portion of the vertical annular cylinder with the increase in Aspect Ratio(A_r).

Fig:2 illustrates the streamlines and isothermal lines distribution inside the porous medium with respect to various values of Radius ratio(R_r). It can be seen that the thickness of the thermal boundary layer decreases with increase in Radius ratio(R_r). The streamlines and isothermal lines move away from the cold wall and reach nearer to the hot wall as Radius ratio(R_r) increases. The isothermal lines are evenly distributed between the two vertical surfaces at smaller Radius ratio(R_r), however it becomes increasingly difficult for thermal energy to penetrate the whole width of porous medium, when Radius ratio(R_r) increases. It also seen that the magnitude of the streamlines decreases with the increase in Radius ratio(R_r).

Fig:3 shows the streamlines and isothermal lines distribution inside the porous medium with respect to various values of viscous dissipation parameter(ε). The circulation of the fluid increases with the increase in viscous dissipation parameter (ε), which is due to the reason that the viscous dissipation parameter (ε) is basically production of heat due to local friction between moving fluid and the solid matrix of the porous medium. The generation of heat due to viscous dissipation parameter (ε) effect increases the temperature inside the medium, which is reflected in terms of greater area of porous medium occupied by increased temperature lines at the upper portion of the vertical annular cylinder. So that the streamlines and isothermal lines move away from the hot wall and reaches nearer to the cold wall of the vertical annular cylinder.

Fig:4 illustrates the streamlines and isothermal lines distribution inside the porous medium with respect to various values of Rayleigh number (Ra). The magnitude of the streamlines increases with the increase in Rayleigh number (Ra). This is due to the reason that the increased Rayleigh number (Ra) promotes the fluid movement due to higher buoyancy force, which in turn allows the convection heat transfer to take dominant position. The increased Rayleigh number (Ra) particularly enhances the heat transfer rate at lower portion of hot and cold walls of vertical annular cylinder respectively. So, the fluid circulation shifts from the lower portion to the upper portion of cold wall of the vertical annular cylinder as Rayleigh number (Ra) increases.

Fig:5 shows the variation of the average Nusselt number$(\overline{Nu})$ at hot wall with respect to Aspect Ratio(A_r) of the vertical annular cylinder. The average Nusselt number$(\overline{Nu})$ decreases with the increase in

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Aspect Ratio(A_r). The temperature difference near the hot wall decreases with increase in viscous dissipation parameter(ε). This happens due to the reason that the viscous dissipation leads to local heat generation, which increases the temperature in the porous medium. As the temperature at the hot wall T_w is constant. The increased temperature of porous medium reduces the temperature difference between the hot wall and the nearby region. Due to this reason the heat transfer from hot wall to the porous medium decreases which results in decreasing the average Nusselt number$(\overline{Nu})$. The effect of viscous dissipation is higher at the lower values of Aspect Ratio(A_r) as compared to the higher values of Aspect Ratio(A_r).

Fig:6 demonstrates the effect of viscous dissipation parameter(ε) on the average Nusselt number$(\overline{Nu})$ for different values of Radius ratio(R_r). The average Nusselt number$(\overline{Nu})$ at hot wall of the vertical annular cylinder increases with the increase in Radius ratio(R_r). Here also it is seen that the effect of viscous dissipation parameter(ε) reduces the average Nusselt number$(\overline{Nu})$ at hot wall of the vertical annular cylinder. This reduction in average Nusselt number$(\overline{Nu})$ at hot wall is more pronounced at higher values of Radius ratio(R_r), with the increase in viscous dissipation parameter(ε).

Fig:7 illustrates the effect of viscous dissipation parameter(ε) on the average Nusselt number$(\overline{Nu})$ for various values of Rayleigh number. It is seen that the average Nusselt number$(\overline{Nu})$ increases linearly with the increase in viscous dissipation parameter(ε). It is also observed that for higher values of Rayleigh number, the average Nusselt number$(\overline{Nu})$ is almost constant with the increase in viscous dissipation parameter(ε).

Conclusion

It is found that more convection heat transfer takes place at the upper portion heated wall of the vertical annular cylinder at higher values of Aspect Ratio(A_r).

The magnitude of streamlines decreases with increase in Radius ratio(R_r). The average Nusselt number $(\overline{Nu})$ increases linearly with the increase in viscous dissipation parameter(ε).

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